Fractals Resource Kit

Introduction

This kit was designed for small and rural libraries who want to hold an inexpensive, STEAM-based program about fractals.

The Colorado State Library (CSL) has launched the *Let's Learn About It!: CSL Big Red Resource Kits* initiative to help small libraries within the state provide STEAM programming for their patrons at little to no cost.

Who is this kit for?

The activities in this kit are primarily for ages 5 and up. Anyone with an interest in art or math will enjoy learning about the magic of fractals! Even the non-mathematical will have fun with the beautiful imagery and the fascinating, mind-bending concepts and patterns (which require next to no math to understand).

Why hold a fractals program?

"Bottomless wonders spring from simple rules...repeated without end." - Benoit Mandelbrot

Fractals are infinitely complex patterns that are "self-similar" - a picture of a fractal looks the same at any magnification. They are created by repeating a simple process over and over. The Cantor set, for example, starts with a line with its middle third removed. Repeat the process with the next two lines, and then the next (and on and on), and you get this:

Cantor set in seven iterations. Image: public domain, via Wikimedia Commons

The process can literally go on to infinity, making this fractal "uncountable."

Since Benoit Mandelbrot coined the term "fractal" in 1975, the field has opened up new ways of describing the natural world with math - something traditional Euclidean geometry can't do.

Fractal geometry is an important, developing field in mathematics with applications from telecommunications to biology to sustainable architecture. Fractals describe the way nature actually looks, from the veins in our lungs to the shape of a mountain range to a floret of broccoli.

The beauty of fractal patterns has influenced art and religion for centuries. Buddhist mandalas and Islamic girih tiles existed long before mathematicians began to describe specific fractal shapes. Traditional African designs in art and the construction of settlements use fractal patterns.

Your program may inspire the next generation of mathematicians, inventors, architects, computer programmers, and artists! The field of fractal geometry is part of every one of those professions.

Feedback

Please help us improve this kit by sharing your comments, experiences, and suggestions. Contact Kit Support (303-866-6900 or email at KitSupport@coloradovirtuallibrary.org) at any point along your journey. When you return the kit, please fill out and include the Feedback Form found at the end of this binder.

We will update this kit's Online Resources page https://cslkits.cvlsites.org/stem-discovery-kits with the ideas and activities your library shares. Help us learn from your experiences and share that learning with others.

Loan period

The loan period for the Fractal Resource Kit is 1 month. If you would like to keep it longer, please contact Kit Support (303-866-6900 or email at KitSupport@coloradovirtuallibrary.org) to make arrangements before the loan period expires. If it is not on hold for another library, we will do our best to extend your loan.

Kit contents are not intended for circulation outside of the library to individual patrons.

Returning the kit

As you prepare to return the kit, verify that it is complete using the Kit Contents checklist in this binder. Be sure to include your completed Feedback Form so that we can improve the experience for everyone.

We know that materials may get damaged in transit or by patrons. If that happens to you, please contact Kit Support (303-866-6900 or email KitSupport@coloradovirtuallibrary.org) and let us know right away!

Kit Contents

Please verify that this resource kit is complete before returning it to: Colorado State Library, Courier Code 912.
You may copy any of the pages from the binder, or download a copy from the red key shaped USB flash drive. You can also find information at https://cslkits.cvlsites.org/stem-discovery-kits.
Contact Kit Support (303-866-6900 or email at KitSupport@coloradovirtuallibrary.org) with questions.

Items

- ❏ 1 1" red resource kit binder containing:
  - ❏ 1 red key-shaped USB flash drive containing printable documents
- ❏ 1 clipboard
- ❏ 12 jumbo magnifiers
- ❏ 1 book: *Mysterious Fractals* (ISBN 9781620916278)
- ❏ 28 white mandala dot painting stencils
- ❏ 2 sets of Dazazzle Mandala Dotting Tools, which include:
  - ❏ 2 plastic paint trays
  - ❏ 2 Charcoal White #558 pencils
  - ❏ 10 acrylic & steel ball stylus tools, various colors and sizes
  - ❏ 16 acrylic rods, various colors and sizes
- ❏ 24 Tessellations animal stencils, assorted designs
- ❏ 2 wooden tile sets containing 352 pieces:
  - ❏ Penrose set #1 (in cloth drawstring Wooden Giraffe bag):
    - ❏ 108 kites with half-circle patterns
    - ❏ 68 dartes with half-circle patterns
  - ❏ Penrose set #2 (in cloth drawstring Wooden Giraffe bag):
    - ❏ 8 decagons
    - ❏ 12 elongated hexagons
    - ❏ 12 bowties
    - ❏ 12 pentagons
    - ❏ 16 darts
    - ❏ 20 kites
    - ❏ 22 elongated rhombuses
    - ❏ 32 diamonds (parallelograms)
    - ❏ 22 obtuse triangles

- ❏ 14 medium isosceles triangles
- ❏ 12 small isosceles triangles
- ❏ 2 tiny isosceles triangles

❏ Super Spirograph 75-Piece Jumbo Kit (50th Anniversary Edition) containing:
- ❏ 31 Spiro-Tracks
- ❏ 21 wheels
- ❏ 2 rings
- ❏ 2 racks
- ❏ .31oz (9g) of reusable putty
- ❏ 3 pens
- ❏ 1 gold die-cast wheel

❏ 1 bin of Puzzellations “In the Garden,” 231 foam tiles

❏ 1 bin of Puzzellations “Kites & Darts,” 105 magnet-backed foam tiles

❏ 3 rubber stamps:
- ❏ 1 Mandelbrot stamp
- ❏ 1 Sierpinski triangle stamp
- ❏ Koch snowflake stamp

Fractal geometry programs

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Introduction

There are many ways to teach people about fractals. The easiest introduction to the mathematical concepts is actually through art, because the appeal of these repeating shapes is obvious, and the simplest fractal shapes are easy (if time-consuming!) to create.

Supplement any artistic activity with a discussion of what makes these shapes mathematically interesting. There are any numbers of introductory videos online you might use to aid discussion or introduce difficult concepts - see the **Other Resources** pages for more information and ideas.

Intended Audience

Recommended for ages 6 and up.

Activity Goals

- Participants understand what a fractal is and can recognize them.
- Participants gain a conceptual understanding of at least one of the mathematical concepts that underlie fractals (per the Common Core Standards for Mathematics).
    - Common Core Standards can specific to fractals can be found in the Fractal Foundation Fractivities sheet (https://bit.ly/2Evl8du). The Colorado Department of Education website (https://bit.ly/2A600Y5) has Colorado's state academic standards.

What staff need to know

"Bottomless wonders spring from simple rules...repeated without end." - Benoit Mandelbrot

Classical geometry describes the neat, ordered world of straight lines, polygons, cubes, and other shapes. It's a useful science. But when was the last time you saw a perfect circle or cone? Classical geometry doesn't apply to most of the natural world!

The "Father of Fractals," Benoit Mandelbrot, wrote "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a

straight line" (*The Fractal Geometry of Nature*, 1982). In fact, most natural objects are formed from fractal shapes (also called "curves"). Fractals are generally defined as complex shapes with a simple recursive construction that are made up of small scale copies of themselves (self-similarity). So if you zoom into a fractal with a magnifying glass or microscope, it will look the same no matter how small you go. Trees, shells, mountains, flowers, clouds, and countless other objects have fractal patterns.

Staff should **be familiar with the basic shapes** you want to introduce during your program. You do *not* need to be able to mathematically describe a fractal curve! Be open to **explore *with* your library users** using the kit activities, and **be able to point the curious to further resources**.

We strongly recommend that you go through the Other Resources page and watch the videos mentioned there before planning your program.

Basic activity ideas

Note: pencils, paints, and non-reusable art supplies are not included in the kit.

- Draw and decorate a fractal shape using spirographs and stencils. There are also dotting tools in the kit that can be used with water-soluble paint. Older participants might try their hand at drawing a Koch snowflake (a shape with finite area, and infinite sides), or a Sierpinski or Apollonian Gasket.
    - Create a display using images of fractals in nature before and after the program to draw participants and for inspiration!
- Use the sets of Penrose tiles to construct repeating patterns, and discuss the history of Islamic art and architecture. Invite an expert from a local mosque or college history department to speak.
- With children ages 5+, have a STEAM-based read aloud, art activity, playtime, and/or snack program:
    - Read from the book in the kit, *Mysterious Patterns*. Pair with other books from your library's collection about geometric shapes or simple math. Pull out your nonfiction children's books that have high-quality photographs of mountain ranges, rivers, lightning, patterns, etc.
    - Purchase a selection of materials from the Supply List "Hold a children's program" list, which includes easily available, low-cost items with patterns that occur in nature. Ask the children to use the magnifying glasses to notice which objects look the same close up as they do from far away. Then serve them the healthy, sciency snacks! (*Please* check with caregivers about food allergies in advance, and wipe down kit supplies afterward!)

    - Art activity: Break out the kit's animal stencils or use the foam tile sets to let kids put together interesting patterns. Mark their artwork and hands using the fractal stamps before they go!

Extension activities - go beyond the kit!

- Teach library users to draw a fractal mandala pattern on a stone or using sand (there are dotting tools in the kit), and discuss the history and meaning of mandalas in Buddhism.
    - Partner with a Buddhist temple and provide a space for the creation of a sand mandala.
- Create string art patterns using string, pins, and cork.
- Build a Sierpinski Gasket in 3D (see the RAFT ideas sheet, included).
- Create a 3D Menger Sponge from folded business cards (see the Maths Craft New Zealand handout, included, plus the Fractal Supply Lists for more details). Fun fact: Menger Sponges have zero volume but infinite surface area!

Check out the Supply Lists for suggested supplies for extension activities.

Promotion

Make a plan for promoting this activity or program. Your plan doesn't have to be extensive, but it should address:

- Age range for attendees (5 and up);
- Budget;
- Any existing library marketing or branding guidelines;
- How you will promote the activity, which could include:
    - Social media
    - Website
    - Email newsletter
    - Printed materials like posters or bookmarks
    - Cross-promoting with partner organizations
    - Local news media
- Creating promotional materials, such as graphics, press releases, or printed materials;
- Timeline

Supply Lists

What you need

Non-reusable art supplies are not included in this kit. Be sure you have everything you need for your program using these checklists!

Basic activity essentials

- ❏ Pencils (regular and colored)
- ❏ Pens (regular and colored)
- ❏ Paper
- ❏ Stamp inkpad (or water-soluble ink and a small sponge)

Optional materials for the basic activity

Art paint supplies:

- ❏ Brushes
- ❏ Fingerpaints, watercolors, etc.
- ❏ Cleaning supplies (paper towels, cups for water, etc.)

Hold a children's program (ages 5+)

Please check with parents for any food allergies before the program!

Fractals in nature:

- ❏ Broccoli
- ❏ Cauliflower
  - ❏ Be sure to get veggie dip so children can snack on the miniature "trees"!
- ❏ Romanesco broccoli cabbage, if available (pictured on the cover of *Mysterious Patterns*)
- ❏ Leaves (tree leaves or lettuce with prominent veins)
- ❏ Fern leaves (real or artificial)
- ❏ Loose cotton batting (to resemble clouds)

Items that are NOT fractals but have interesting patterns and are also delicious:

- ❏ Carrot sticks
- ❏ Grapes

- ❏ Strawberries (whole)
- ❏ Kiwi, sliced to reveal the pattern in the center
- ❏ Pineapple, whole and/or sliced. Give each child a look at the patterned skin with their magnifying glasses.

Extension activities - how to go beyond the basics

➽ Teach library users to draw a fractal mandala pattern on a stone or using sand, and discuss the history and meaning of mandalas in Buddhism.

- ❏ Partner with a Buddhist temple and provide a space and materials for the creation of a sand mandala.

➽ Create string art patterns using string, pins, and cork.

➽ Build a Sierpinski Gasket in 3D (see the RAFT ideas sheet, included).

➽ Put together a Menger Sponge (see the Maths Craft New Zealand instructions, included.)

Additional library supplies needed for extension activities

➽ Creating sand mandalas

- ❏ White glue
- ❏ Multi-colored craft sand
- ❏ Glitter
- ❏ Cardstock or construction paper
- ❏ Pencils (regular and colored)
- ❏ Knob brushes (for brushing away mandalas made with no glue - optional)

➽ Painting fractals on stones

- ❏ Small stones (white and black stones are available to purchase online)
- ❏ Crayola Rock Painting Kit or similar supplies
- ❏ Washable paints

➽ Creating fractal string art

- ❏ Multi-colored embroidery floss/twine/yarn
- ❏ Cork tiles
- ❏ Long, straight push pins
- ❏ Printed fractal/mandala or other patterns to follow (available online)

➳ Creating a Sierpinski Gasket in 3D (see the RAFT ideas sheet, included in this kit)

- ❏ Glue gun(s)
- ❏ Hot glue
- ❏ Pony beads

➳ Creating a Menger Sponge

Varying amounts of business cards are needed for smaller or larger constructions: a single cube requires 6 cards, plus printed "cladding" that will give the illusion of your construction being made of infinitely smaller cubes). Check out the Maths Craft New Zealand instructions, included in the kit, and the MegaMenger project website (link below) for more free printables. MegaMenger also has a free printable pattern for those with 3D printers.

- ❏ Level 1: 120 cards
- ❏ Level 2: 400 cards
- ❏ Level 3: 8,000 cards
- ❏ "Cladding": printable Menger Cards can be found at: http://megamenger.com/

Feedback Form

Instructions

Please help us improve this Fractal Resource Kit by completing either this paper feedback form or the online version, found at https://cslkits.cvlsites.org/stem-discovery-kits. If you are completing the paper form, please return it with the kit.

Part 1: Evaluation

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
I am satisfied that the resources in this kit met the library's needs.					
This resource will help improve library services to the public					
The loan period was adequate.					
I would recommend this kit to another library.					

Which parts of the kit did you find useful, and why?

Were there any parts of the kit that your library didn't use ? Why not?

How can we improve this kit?

Part 2: How Your Library Used the Kit

Please describe any of the activities or programs you used from this kit. Be as specific as possible. Include links, comments, and/or evaluation data when available.

** Want another way to keep the conversation going and share your experiences with others? Visit our forum on the CSL Resource Kit website (https://cslkits.cvlsites.org/discuss/) and share what you are doing there.**

Other Resources

Where else you can find this information

We have included a red flash key with this kit. It is a USB flash drive containing these pages from the binder and other resources. Updated resources can be found at https://cslkits.cvlsites.org/stem-discovery-kits.

General

WEBSITES

➳ Colorado Academic Standards

http://www.cde.state.co.us/standardsandinstruction/2009standards

Plenty of useful information if you're trying to partner with a local school or with student-age library users. Check out the Online Sortable Tool and try searching "patterns" within the Mathematics or Science sections.

➳ Denver Art Museum > Hayagriva Mandala

https://denverartmuseum.org/edu/object/hayagriva-sand-mandala

Visit the site to see a sand mandala gifted to DAM by the Seraje Monastery in Tibet. There are excellent teaching resources available at the link, including a detailed lesson plan, videos of the mandala's creation, and a printable coloring page.

➳ MegaMenger Project

http://megamenger.com/

The Menger Sponge is a mind-bending 3D fractal shape, and this site supported by the University of London has free downloadable instructions on creating a Menger Sponge using business cards. There are lots of pictures of completed projects worldwide, plus templates to create your own "cladding" to give your creation more layers. Check out the Activities & Programs for more ideas.

➳ RAFT: Resource Area for Teaching > Freaky Fractals

http://raft.education/activity_kits/freaky-fractals/

Purchase preassembled kits from RAFT to use in your program (membership required), or simply download their idea sheet (also available in this kit). Links to various teaching standards can be found here.

➽ TopIslamic.com > Islamic Art for Kids 15 Colouring Pages

https://topislamic.com/islamic-art-for-kids-download/

Free coloring pages showing traditional Islamic art, provided by blogger and teacher Salah Ad Deen.

➽ University of Kansas School of Education > Fractals for Fun: Teaching Kids Patterns in Nature

A great introductory site that has gathered plenty of useful links to descriptions, images, games, and other activities.

https://educationonline.ku.edu/articles/teaching-kids-patterns-in-nature

VIDEOS

➽ Doodling In Math: Infinity Elephants: Khan Academy

https://www.khanacademy.org/math/math-for-fun-and-glory/vi-hart/doodling-in-math/v/doodling-in-math-class-infinity-elephants

A challenge for math class doodlers! This video is part of a mesmerizing series demonstrating doodling in math, and it's also funny and clever.

➽ Fractals and the Art of Roughness: Benoit Mandelbrot TED2010

https://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness

Fractals discussed by the mathematician who first named them.

➽ The Fractals at the Heart of African Designs: Ron Eglash TEDGlobal 2007

https://www.ted.com/talks/ron_eglash_on_african_fractals

Ethno-mathematician Eglash, author of the book *African Fractals* (Rutgers UP, 2002) talks about his research into African design.

➽ Koch Snowflake Fractal: Khan Academy

https://www.khanacademy.org/math/geometry-home/geometry-volume-surface-area/koch-snowflake/v/koch-snowflake-fractal

For those who want to know the math, Sal Khan explains it in this short video series. Learn how "A shape that has an infinite perimeter but finite area" is mathematically possible.

➽ Stories About Nature: Michael Frame at TEDxYale 2013

https://www.youtube.com/watch?v=bz8NJ7ZVXwQ

Lots of great visual examples of fractal designs.

➽ What Is a Fractal (And What Are They Good For?): Khan Academy/MIT

https://www.khanacademy.org/partner-content/mit-k12/mit-math/v/what-is-a-fractal-and-what-are-they-good-for

A concise introduction that talks about how fractals have been used in technology like antennas, and demonstrates a Koch snowflake.

APPS

➽ Frax - Immersive Fractals

Review in the *Smithsonian Magazine*: "A New App Turns Fractals Into Ornate Art". Short URL: https://bit.ly/2EtqohC

A $.99 app built on the Mandelbrot and Julia set equations, it allows users to play and be immersed in the art of fractals. For IOS.

➽ Mandala Maker: symmetry doodle

Draw and color intricate mandala patterns with this app for Android and IOS.

BOOKS

➽ *Fractals: A Very Short Introduction*. Kenneth Falconer. Oxford University Press, 2013. 9780199675982.

> The introduction is accessible and has good drawings of fractals and the motifs/generators that make them up. Your patrons who want to go beyond the Khan Academy math videos will appreciate the deeper math Falconer includes.

➽ *Fractals: A Graphic Guide*. Nigel Lesmoir-Gordon, Will Rood & Ralph Edney. Icon Books, 2009.

Some of the art is downright terrible, but this is an accessible introduction that will give people who want to know a little more just the right amount of information about what fractals are, how they were first described, and how they are used today.

➳ *Mysterious Patterns: Finding Fractals in Nature*. Sarah C. Campbell and Richard Campbell. Boyds Mills Press, 2014. ISBN 9781620916278.

Included in kit. If you only read one book about fractals, this one will give you just the right start, with lots of pictures to illustrate the ideas.

➳ *Patterns in Nature: Why the Natural World Looks the Way It Does*. Philip Ball. University of Chicago Press, 2016. ISBN 9780226332420

Gorgeous, full-color photographs fill this book. Only chapter 2 is about fractals, and the rest of the book is about other types of patterns found in nature. Accessible and eye-catching.

YouTube

➳ There are more videos on this topic than can be included here - many include mesmerizing imagery that will take you down the rabbit hole for hours.

If your patrons have dipped their toes in and can't wait to learn more, below are a set of internet search terms that will turn up beautiful images and fascinating mathematical history.

Search terms for inspiration

Cantor set

Devil's staircase (Cantor set)

Eulerian path/Eulerian circuit

Fournier fractal (shows that an infinite universe need not be equally bright in all directions, if it has a fractal structure - answer to Olbers' Paradox)

Indra's net

Julia set/Fatou set (couldn't be adequately visualized until computers caught up - though the math was described during WWI by Gaston Julia and Pierre Fatou)

Lichtenberg figure

Lorenz attractor/system

Mandelbrot set

Pascal's triangle (contains the Sierpinski gasket - color the odd numbers black, the even ones white to make the fractal easy to see)

Peano curve

Penrose tiling (see also: Darb-i-Imam shrine in Isfahan, Iran, built in 1453)

Saturn's rings

Symmetry in nature

Tessellation

Turing patterns

Viscous fingering